

Fractions and decimals

Adding or subtracting two fractions with different denominators

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

The product of two fractions multiplies straight across.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Comparing two fractions with positive denominators

$$\frac{a}{b} > \frac{c}{d} \iff ad > bc$$

The original whole equals the final leftover amount divided by the product of the remaining shares.

$$W = \frac{L}{(1-f_1)(1-f_2)\cdots(1-f_n)}$$

Adding the same amount k to both numerator and denominator moves a positive fraction toward 1.

$$\frac{a+k}{b+k}$$

Multiplying the numerator by m and the denominator by n multiplies the fraction by $\frac{m}{n}$.

$$\frac{a \times m}{b \times n} = \frac{a}{b} \times \frac{m}{n}$$

For positive fractions, the larger fraction has the smaller reciprocal.

$$\frac{a}{b} > \frac{c}{d} \implies \frac{b}{a} < \frac{d}{c} \quad (a, b, c, d > 0)$$

The number of decimal places a terminating fraction has, given its reduced denominator

$$\text{decimal places} = \max(a, b)$$

for a denominator of $2^a \times 5^b$

The value of a digit in the k th place after the decimal point

$$d \times 10^{-k}$$

A decimal $0.d_1d_2\ldots d_n$ with unknown digits, expressed as a linear combination of those digits

$$0.d_1d_2\ldots d_n = \frac{10^{n-1}d_1 + 10^{n-2}d_2 + \cdots + d_n}{10^n}$$

Multiplying decimals

$$a \times b = \frac{A \times B}{10^{d_a+d_b}}$$

Multiplying two numbers in scientific notation

$$(a \times 10^m) \times (b \times 10^n) = (a \times b) \times 10^{m+n}$$

Mixed scales

$$(a \times 10^m)(b \times 10^n) = ab \times 10^{m+n},$$

$$\frac{a \times 10^m}{b \times 10^n} = \frac{a}{b} \times 10^{m-n}$$

Number properties

The sign of a quotient matches the sign of its corresponding product

$$\text{sign}\left(\frac{x}{y}\right) = \text{sign}(x \cdot y)$$

The sign of a power depends on whether the exponent is even or odd.

If n is even,
 $x^n > 0$ (for $x \neq 0$). If n is odd,
 x^n has the same sign as x .

Difference between a two-digit number and its reverse

$$(10a + b) - (10b + a) = 9(a - b)$$

Difference between a three-digit number and its reverse

$$(100a + 10b + c) - (100c + 10b + a) = 99(a - c)$$

A condition relating the tens digit to the units digit

$$a = k \cdot b$$

When the divisor exceeds the dividend, the quotient is zero and the remainder equals the dividend.

$$n = d \cdot q + r \implies q = 0 \text{ and } r = n$$

The remainder from a decimal quotient

$$r = \left(\frac{n}{d} - q\right) \times d$$

Powers

$$E = k \cdot L + r \implies a^E \equiv c_r \pmod{d}$$

A is a factor of b when b equals a times an integer k

$$b = a \cdot k$$

One less than a square factors into two consecutive integers

$$n^2 - 1 = (n - 1)(n + 1)$$

The prime factorization of an integer greater than 1

$$n = p^a q^b$$

The number of positive factors of an integer from its prime factorization

$$\text{Number of factors} = (a + 1)(b + 1)$$

The power of a prime p in the factorial of n

$$\left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \dots$$

If $\gcd(a, b) = g$, write $a = gm$ and $b = gn$ with m and n coprime.

$$a = gm, b = gn, \gcd(m, n) = 1$$

The number of integers in an inclusive range from a first value to a last value

$$\text{count} = \text{last} - \text{first} + 1$$

For any evenly spaced set, the sum equals the average of the first and last terms times the count of terms.

$$\text{sum} = \frac{\text{first} + \text{last}}{2} \times \text{count}$$

The algebraic representation of k consecutive even or odd integers starting from n

$$n, n + 2, n + 4, \dots, n + 2(k - 1)$$

Exponents and roots

Product rule

$$a^m \times a^n = a^{m+n}$$

Quotient rule

$$\frac{a^m}{a^n} = a^{m-n}$$

Power rule

$$(a^m)^n = a^{mn}$$

Exponent rules

$$a^m \times a^n = a^{m+n} \text{ and } \frac{a^m}{a^n} = a^{m-n} \text{ and } (a^m)^n = a^{mn}$$

A power of a product splits across the factors, and a shared exponent combines into a single base.

$$(ab)^n = a^n \times b^n$$

$$a^n \times b^n = (ab)^n$$

Rewrite a composite base as a power of its prime factor before applying exponent rules.

$$a = p^k \implies a^m = (p^k)^m = p^{km}$$

Factoring a sum of powers with the same base

$$a^m + a^{m+n} = a^m(1 + a^n)$$

The three exponent extensions

$$a^0 = 1 \text{ and } a^{-n} = \frac{1}{a^n} \text{ and } a^{m/n} = \sqrt[n]{a^m}$$

A negative exponent means take the reciprocal of the base and make the exponent positive.

$$a^{-n} = \frac{1}{a^n}$$

A fraction raised to a negative power flips the fraction and makes the exponent positive.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

A fractional exponent converts to a root, where the denominator becomes the index of the root.

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

For any base between 0 and 1, increasing the exponent decreases the value.

$$0 < a < 1 \implies a^3 < a^2 < a$$

When two powers of the same base are equal, the exponents are equal.

$$a^x = a^y \implies x = y \quad (a \neq 0, 1, -1)$$

Common base

$$p^a = p^b \implies a = b$$

The square root of a product is the product of the square roots.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

Combine radicals that share the same radicand by adding or subtracting their coefficients.

$$a\sqrt{x} + b\sqrt{x} = (a + b)\sqrt{x}$$

The principal square root of a squared variable is its absolute value.

$$\sqrt{x^2} = |x|$$

The cube root of a real number cubed

$$\sqrt[3]{x^3} = x$$

Algebra

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Difference of squares

$$a^2 - b^2 = (a + b)(a - b)$$

Linear equation

$$ax + b = 0 \implies x = -\frac{b}{a}$$

The solution to a linear equation in standard form

$$x = \frac{c - b}{a}$$

Collecting terms for a linear equation with variables on both sides

$$ax + b = cx + d \implies x(a - c) = d - b$$

A system of two independent linear equations in two unknowns has a unique solution

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

Substitute an isolated variable into the other equation

$$x = \frac{c - by}{a}$$

$$\implies a_2\left(\frac{c - by}{a}\right) + b_2y = c_2$$

Two-type mixture from total count and total value

$$x + y = n, p_1x + p_2y = V$$

A target expression is determined when it is a linear combination of the given equations.

$$a \cdot E_1 + b \cdot E_2 = T$$

Divisibility shortcut

$$y = \frac{c - ax}{b}$$

The distributive property

$$a(b + c) = ab + ac$$

A negative sign before parentheses reverses every term inside.

$$-(a - b + c) = -a + b - c$$

The product of two binomials

$$(a + b)(c + d) = ac + ad + bc + bd$$

Factoring a sum by pulling out the greatest common factor

$$ab + ac = a(b + c)$$

The three special products

$$(a + b)^2 = a^2 + 2ab + b^2,$$

$$(a - b)^2 = a^2 - 2ab + b^2,$$

$$(a + b)(a - b) = a^2 - b^2$$

The square of a binomial

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

The sum of two squares equals the square of their sum minus twice their product.

$$a^2 + b^2 = (a + b)^2 - 2ab$$

Simplifying an algebraic fraction by cancelling a shared factor

$$\frac{ax}{ay} = \frac{x}{y} \text{ where } a \neq 0 \text{ and } y \neq 0$$

The sum of two fractions with unlike denominators

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Multiplying and dividing two algebraic fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \text{ and } \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Standard form of a quadratic equation

$$ax^2 + bx + c = 0$$

A product relation with a dependency becomes a quadratic in one variable

$$xy = k \text{ and } y = ax + b$$

$$\implies x(ax + b) = k$$

The product of two consecutive integers

$$n(n + 1) = k$$

Revenue as the product of price and quantity

$$R = P \times Q$$

A quadratic with roots r and s

$$(x - r)(x - s) = 0$$

Sum and product of the roots of $ax^2 + bx + c = 0$

$$r_1 + r_2 = -\frac{b}{a} \text{ and } r_1 \times r_2 = \frac{c}{a}$$

Sum of the squares of the roots

$$r^2 + s^2 = (r + s)^2 - 2rs$$

Discriminant

$$\Delta = b^2 - 4ac$$

$\Delta > 0$: two real roots

$\Delta = 0$: one real root

$\Delta < 0$: no real roots

Inequalities and absolute value

Flip on multiply

$$a < b, c < 0 \implies ac > bc$$

Triangle inequality

$$|a + b| \leq |a| + |b|$$

The distance between two points on a number line

$$|a - b| = |b - a|$$

The sign of a variable revealed by its absolute value

$$|x| = x \implies x \geq 0, |x| = -x$$

$$\implies x \leq 0, \sqrt{x^2} = |x|$$

Bounds on a combined absolute value

$$|x + y| \leq |x| + |y|$$

$$|x - y| \geq ||x| - |y||$$

An absolute value equation splits into two linear equations.

$$|cx - d| = b \implies cx - d = b$$

$$\text{or } cx - d = -b$$

A value within b units of a

$$|x - a| < b \text{ becomes } a - b < x < a + b$$

The value of x that is equidistant from a and b

$$x = \frac{a + b}{2}$$

A range from p to q written as an absolute value inequality $ x - \frac{p+q}{2} \leq \frac{q-p}{2}$
The ranges where an operation changes a number, found by factoring the inequality $x^2 > x \implies x(x-1) > 0 \implies x > 1$ or $x < 0$
The four test ranges for number properties $x < -1, -1 < x < 0, 0 < x < 1, x > 1$

Percents and interest
Successive percent change $(1 + r_1)(1 + r_2) - 1$
Original value before a percent change of p percent $O = \frac{N}{1 + \frac{p}{100}}$
The overall multiplier for a value that changes by p% and then by q% Overall multiplier = $\left(1 + \frac{p}{100}\right)\left(1 + \frac{q}{100}\right)$
Net multiplier for a rise and fall of the same percent $(1 + \frac{p}{100})(1 - \frac{p}{100}) = 1 - \frac{p^2}{10000}$
The percent r that undoes a p% change $(1 + \frac{p}{100})(1 + \frac{r}{100}) = 1$
Overall multiplier for three successive percent changes $(1 + \frac{p}{100})(1 + \frac{q}{100})(1 + \frac{r}{100})$
Overall multiplier for a product when two factors change by percents $M = \left(1 + \frac{p}{100}\right)\left(1 + \frac{q}{100}\right)$
A quantity p percent more than a base B $A = B\left(1 + \frac{p}{100}\right)$
For chained comparisons, multiply the decimal multipliers. $A = m_1 \times m_2 \times C$
Interest earned $I = P \times r \times t$
Total interest from two split deposits $I = P_1r_1t_1 + P_2r_2t_2$
Final balance with compound interest balance = $P\left(1 + \frac{r}{n}\right)^{nt}$
The value after a constant factor change over several periods $V = V_0 \times r^n$
Final value after decay $V = V_0 \times \left(1 - \frac{r}{100}\right)^n$
Ratios
Ratio to fraction $\frac{a}{a+b}$
Cross multiplication $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$
New ratio from known changes to both parts $\frac{ax+p}{bx+q} = \frac{c}{d}$
Direct proportion keeps the quotient constant, and inverse proportion keeps the product constant. Direct: $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ Inverse: $x_1y_1 = x_2y_2$
Matching units in matching positions $\frac{a}{b} = \frac{c}{d} \implies a \times d = b \times c$
The product of two inversely proportional quantities stays constant across two states. $x_1y_1 = x_2y_2$

Rates and work
Combined work rate $\frac{1}{t_{\text{combined}}} = \frac{1}{t_1} + \frac{1}{t_2}$
Average speed (equal distances) $\frac{2 \times s_1 \times s_2}{s_1 + s_2}$
Distance equals rate multiplied by time $d = rt$
Distance equals rate multiplied by time, but only when their time units match. $d = r \times t$
Average speed for a multi-leg trip Average Speed = $\frac{\text{Total Distance}}{\text{Total Time}}$
The ratio of times for the same distance is the inverse of the ratio of rates. $\frac{t_1}{t_2} = \frac{r_2}{r_1}$
The ratio of distances for the same time equals the ratio of rates. $\frac{d_1}{d_2} = \frac{r_1}{r_2}$
Time to close the gap when moving toward each other $t = \frac{\text{gap}}{r_1 + r_2}$
Time for a single leg at speed r over distance d $t = \frac{d}{r}$
Total work across phases with different rates $W = r_1t_1 + r_2t_2 + \dots + r_nt_n$
One completed job from a group of workers $(r_A + r_B + \dots + r_n) \times t = 1$
Combined rate equals the sum of individual rates $\frac{1}{x} + \frac{1}{x+d} = \frac{1}{c}$
Relationship between rate ratio and time ratio $\frac{r_A}{r_B} = \frac{t_B}{t_A}$
The unknown rate is the combined rate minus the known rate. $r_{\text{unknown}} = r_{\text{combined}} - r_{\text{known}}$
Mixtures and weighted averages
The combined average, given each group's average and size combined average = $\frac{a_1n_1 + a_2n_2 + \dots}{n_1 + n_2 + \dots}$
The ratio of group sizes is the inverse of the ratio of their distances from the combined average. $\frac{n_A}{n_B} = \frac{d_2}{d_1}$
The size of a group from its ratio share and the total count $n_A = \left(\frac{r_A}{r_A + r_B}\right) \times N$
The final concentration is the total amount of the ingredient divided by the total volume. $c = \frac{c_1v_1 + c_2v_2}{v_1 + v_2}$
Combined concentration of two solutions $c_1v_1 + c_2v_2 = c(v_1 + v_2)$
The ratio of the weights is determined by the distances from the combined average to the group averages. $\frac{w_A}{w_B} = \frac{ C - B }{ C - A }$
Statistics
Mean $\sum \frac{x_i}{n}$

Weighted average $\frac{\sum w_ix_i}{\sum w_i}$
Standard deviation $\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$
The position of the median in an ordered list of n values median position = $\frac{n+1}{2}$
The range of a set of values Range = Maximum – Minimum
Overlapping sets
Two overlapping groups total = $A + B - \text{both} + \text{neither}$
Total members across three groups, given the exactly-two count, the all-three count, and the neither count total = $A + B + C - (\text{exactly two}) - 2 \times (\text{all three}) + \text{neither}$
Counting, probability, functions and sequences
Finding an identity value for a custom operation $a \otimes e = a$
The term at position n in a sequence that adds the same amount each step $a_n = a_1 + (n - 1)d$
The term at position n in a sequence that multiplies by the same ratio each step $a_n = a_1r^{n-1}$
The common difference of an arithmetic sequence from two known terms $d = \frac{a_m - a_k}{m - k}$
The sum of a set of evenly spaced terms $S_n = \frac{n(a_1 + a_n)}{2}$
The constant added in an arithmetic sequence; The constant multiplied in a geometric sequence $d = a_{n+1} - a_n$ $r = \frac{a_{n+1}}{a_n}$
The nth term as a function of earlier terms $a_n = f(a_{n-1}, \dots)$
Each term is the rule applied to the previous term $a_n = f(a_{n-1})$
The next term depends on the two previous terms. $a_n = f(a_{n-1}, a_{n-2})$
The inverted recursive rule to find the previous term $a_{n-1} = f^{-1}(a_n)$
The position of a target term within a repeating cycle $r = n \bmod p$
Total outcomes when choosing one option from each independent stage total = $a \times b \times c \times \dots$
Total outcomes when repetition is allowed across k slots $n \times n \times \dots \times n = n^k$
Total arrangements of distinct items in sequence without repetition $n \times (n - 1) \times (n - 2) \times \dots$
Outcomes in either of two disjoint cases A or B $ A \cup B = A + B $
Allowed outcomes by the complement method $ A = T - A^c $

The number of ways to arrange r of n distinct objects in order $\frac{n!}{(n-r)!}$
The number of arrangements of n distinct objects where a specific group of k objects must be adjacent $(n - k + 1)! \times k!$
Non-adjacent items $n! \times \binom{n+1}{k} \times k!$
The number of distinct arrangements of n objects where some are identical $\frac{n!}{n_1! \times n_2! \times \dots \times n_k!}$
Distinct circular arrangements of n distinct objects $(n - 1)!$
The number of ways to choose r items from n items when order does not matter $\binom{n}{r} = \frac{n!}{r!(n-r)!}$
The number of unordered pairs among n items $\binom{n}{2} = \frac{n(n-1)}{2}$
Ordered selections of r items chosen from n $_nP_r = \frac{n!}{(n-r)!} = \binom{n}{r} \times r!$
Fewest draws to guarantee n matching items of any kind when k kinds have enough supply $k(n - 1) + 1$
The probability of event A or event B occurring $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
The probability that all independent events occur $P(\text{all}) = P(A) \times P(B) \times P(C) \times \dots$
Probability of at least one success among independent events $P(\text{at least one}) = 1 - P(\text{none})$
Probability of a sequence of draws without replacement $P(A \text{ and } B) = P(A) \times P(B \mid A)$
The probability of A given B $P(A B) = \frac{P(A \text{ and } B)}{P(B)}$
Two events A and B are independent when the conditional probability equals the unconditional probability $P(A B) = P(A)$
Data Insights
The AD/BCE branch after testing statement (1) alone (1) sufficient $\rightarrow$ A or D; (1) insufficient $\rightarrow$ B, C, or E
A linear equation in one variable yields exactly one solution. $ax + b = c \Rightarrow x = \frac{c-b}{a}, a \neq 0$
Percent change equals the difference between the end and start values divided by the start value. percent change = $\frac{\text{end value} - \text{start value}}{\text{start value}}$
Count of rows passing every condition, where each F_i is the set of rows that pass one filter count = $ F_1 \cap F_2 \cap \dots \cap F_n $
Weighted average of values with corresponding weights Weighted average = $\frac{\sum (\text{value} \times \text{weight})}{\sum \text{weights}}$
Linked equations $f(x, y) = 0$